Combination and Composition of Functions

Domain of Rational and Square Root Functions

Example 1:

Suppose \( y = \frac{1}{x} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>y is defined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>( 1/(-5) = -0.2 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>-4</td>
<td>( 1/(-4) = -0.25 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>-3</td>
<td>( 1/(-3) = -0.333333 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>-2</td>
<td>( 1/(-2) = -0.5 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>-1</td>
<td>( 1/(-1) = -1 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>0</td>
<td>( 1/(0) = ) Undefined</td>
<td>( y ) is undefined</td>
</tr>
<tr>
<td>1</td>
<td>( 1/(1) = 1 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>2</td>
<td>( 1/(2) = 0.5 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>3</td>
<td>( 1/(3) = 0.333333 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>4</td>
<td>( 1/(4) = 0.25 )</td>
<td>( y ) is defined</td>
</tr>
<tr>
<td>5</td>
<td>( 1/(5) = 0.2 )</td>
<td>( y ) is defined</td>
</tr>
</tbody>
</table>

If \( x = 0 \), then \( y = 1/0 \) is undefined.

If \( x \neq 0 \), then \( y \) is defined.

\[ y \text{ is defined} \quad \oplus \quad y \text{ is defined} \]

\[ 0 \]

Domain of the function \( y = \frac{1}{x} \) is \( (-\infty, 0) \cup (0, \infty) \).
Example 2:

Suppose \( y = \frac{1}{x - 2} \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>y is defined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>1/(-7) = -0.142857</td>
<td>y is defined</td>
</tr>
<tr>
<td>-4</td>
<td>1/(-6) = -0.166667</td>
<td>y is defined</td>
</tr>
<tr>
<td>-3</td>
<td>1/(-5) = -0.2</td>
<td>y is defined</td>
</tr>
<tr>
<td>-2</td>
<td>1/(-6) = -0.25</td>
<td>y is defined</td>
</tr>
<tr>
<td>-1</td>
<td>1/(-3) = -0.333333</td>
<td>y is defined</td>
</tr>
<tr>
<td>0</td>
<td>1/(-2) = -0.5</td>
<td>y is defined</td>
</tr>
<tr>
<td>1</td>
<td>1/(-1) = -1</td>
<td>y is defined</td>
</tr>
<tr>
<td>2</td>
<td>1/(0) = undefined</td>
<td>y is undefined</td>
</tr>
<tr>
<td>3</td>
<td>1/(1) = 1</td>
<td>y is defined</td>
</tr>
<tr>
<td>4</td>
<td>1/(2) = 0.5</td>
<td>y is defined</td>
</tr>
<tr>
<td>5</td>
<td>1/(3) = 0.3333333</td>
<td>y is defined</td>
</tr>
</tbody>
</table>

If \( x = 2 \), then \( y = 1/0 \) is undefined.

If \( x \neq 2 \), then \( y \) is defined.

Domain of the function \( y = \frac{1}{x - 2} \) is \( (-\infty, 2) \cup (2, \infty) \).
Example 3:

Suppose \( y = \frac{x}{(x-2)(x+4)} \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-0.2592593</td>
<td>y is defined</td>
</tr>
<tr>
<td>-6</td>
<td>-0.375</td>
<td>y is defined</td>
</tr>
<tr>
<td>-5</td>
<td>-0.7142857</td>
<td>y is defined</td>
</tr>
<tr>
<td>-4</td>
<td>Undefined</td>
<td>y is undefined</td>
</tr>
<tr>
<td>-3</td>
<td>0.6</td>
<td>y is defined</td>
</tr>
<tr>
<td>-2</td>
<td>0.25</td>
<td>y is defined</td>
</tr>
<tr>
<td>-1</td>
<td>0.11111111</td>
<td>y is defined</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>y is defined</td>
</tr>
<tr>
<td>1</td>
<td>-0.2</td>
<td>y is defined</td>
</tr>
<tr>
<td>2</td>
<td>Undefined</td>
<td>y is undefined</td>
</tr>
<tr>
<td>3</td>
<td>0.42857143</td>
<td>y is defined</td>
</tr>
<tr>
<td>4</td>
<td>0.25</td>
<td>y is defined</td>
</tr>
<tr>
<td>5</td>
<td>0.18518519</td>
<td>y is defined</td>
</tr>
</tbody>
</table>

If \( x = -4 \), then \( y \) is undefined. If \( x = 2 \), then \( y \) is undefined.

If \( x \neq -4 \) and \( x \neq 2 \), then \( y \) is defined.

\[
\text{Domain of the function } y = \frac{x}{(x-2)(x+4)} \text{ is } (-\infty, -4) \cup (-4, 2) \cup (2, \infty).
\]
Example 4:

Suppose \( y = \frac{x}{x^2 + 4x + 3} \). Then \( y = \frac{x}{(x+3)(x+1)} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y ) is defined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>-0.2916667</td>
<td>y is defined</td>
</tr>
<tr>
<td>-6</td>
<td>-0.4</td>
<td>y is defined</td>
</tr>
<tr>
<td>-5</td>
<td>-0.625</td>
<td>y is defined</td>
</tr>
<tr>
<td>-4</td>
<td>-1.3333333</td>
<td>y is defined</td>
</tr>
<tr>
<td>-3</td>
<td>undefined</td>
<td>y is undefined</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
<td>y is defined</td>
</tr>
<tr>
<td>-1</td>
<td>undefined</td>
<td>y is undefined</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>y is defined</td>
</tr>
<tr>
<td>1</td>
<td>0.125</td>
<td>y is defined</td>
</tr>
<tr>
<td>2</td>
<td>0.13333333</td>
<td>y is defined</td>
</tr>
<tr>
<td>3</td>
<td>0.125</td>
<td>y is defined</td>
</tr>
<tr>
<td>4</td>
<td>0.11428571</td>
<td>y is defined</td>
</tr>
<tr>
<td>5</td>
<td>0.10416667</td>
<td>y is defined</td>
</tr>
</tbody>
</table>

If \( x = -3 \), then \( y \) is undefined. If \( x = -1 \), then \( y \) is undefined.

If \( x \neq -3 \) and \( x \neq -1 \), then \( y \) is defined.

\[ y \] is defined \hspace{1cm} y \] is defined \hspace{1cm} y \] is defined

\(-3\) \hspace{2cm} \(-1\)

Domain of the function \( y = \frac{x}{x^2 + 4x + 3} \) is \((-\infty, -3)\cup(-3, -1)\cup(-1, \infty)\).
Example 5:

Suppose \( y = \frac{x}{x^2 - 9} \). Then \( y = \frac{x}{(x + 3)(x - 3)} \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-6</td>
<td>-0.2222222222</td>
</tr>
<tr>
<td>-5</td>
<td>-0.3125</td>
</tr>
<tr>
<td>-4</td>
<td>-0.571428571</td>
</tr>
<tr>
<td>-3</td>
<td>undefined</td>
</tr>
<tr>
<td>-2</td>
<td>0.4</td>
</tr>
<tr>
<td>-1</td>
<td>0.125</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>-0.125</td>
</tr>
<tr>
<td>2</td>
<td>-0.4</td>
</tr>
<tr>
<td>3</td>
<td>undefined</td>
</tr>
<tr>
<td>4</td>
<td>0.571428571</td>
</tr>
<tr>
<td>5</td>
<td>0.3125</td>
</tr>
<tr>
<td>6</td>
<td>0.2222222222</td>
</tr>
</tbody>
</table>

If \( x = -3 \), then \( y \) is undefined. If \( x = 3 \), then \( y \) is undefined.

If \( x \neq -3 \) and \( x \neq 3 \), then \( y \) is defined.

\[ \text{Domain of the function } y = \frac{x}{x^2 - 9} \text{ is } (-\infty, -3) \cup (-3, 3) \cup (3, \infty). \]
Example 6:

Find the domain of the function \( y = \frac{x}{4x^3 - 2x^2 + 6x - 3} \)

\[
y = \frac{x}{2x^2(2x-1) + 3(2x-1)} = \frac{x}{(2x^2 + 3)(2x-1)}
\]

Note: \( 2x^2 + 3 > 0 \)

Set \( 2x - 1 = 0 \)
\[
2x - 1 + 1 = 0 + 1
\]
\[
x = 1
\]
\[
\frac{2x}{2} = \frac{1}{2}
\]
\[
x = \frac{1}{2}
\]

Thus, if \( x = 1/2 \), \( y \) is undefined

If \( x \neq 1/2 \), then \( y \) is defined.

y is defined \hspace{2cm} y is defined

Domain of the function \( y = \frac{x}{4x^3 - 2x^2 + 6x - 3} \) is \((-\infty, 1/2) \cup (1/2, \infty)\).
Example 7:

Find the domain of the function \( y = \frac{1}{4 + x} - \frac{2}{x - 3} \)

If \( x = -4 \), \( y = \frac{1}{4 + x} - \frac{2}{x - 3} = \frac{1}{4 + (-4)} - \frac{2}{(-4) - 3} = \text{undefined} + \frac{2}{7} = \text{undefined} \)

If \( x = 3 \), \( y = \frac{1}{4 + x} - \frac{2}{x - 3} = \frac{1}{4 + (3)} - \frac{2}{(3) - 3} = \frac{1}{7} + \text{undefined} = \text{undefined} \)

Thus, if \( x = -4 \) or \( x = 3 \), \( y \) is undefined

If \( x \neq -4 \) and \( x \neq 3 \), then \( y \) is defined.

\[ \begin{array}{ccc} y \text{ is defined} & y \text{ is defined} & y \text{ is defined} \\ -4 & \text{ } & 3 \end{array} \]

Domain of the function \( y = \frac{1}{4 + x} - \frac{2}{x - 3} \) is \((-\infty, -4) \cup (-4, 3) \cup (3, \infty)\).
Example 8:

Find the domain of the function \( y = \frac{1}{x-5} + 5 - \frac{2}{x^2 + 3} \)

If \( x = 5 \),

\[
y = \frac{1}{5 - 5} + 5 - \frac{2}{5^2 + 3} = \frac{1}{0} + 5 - \frac{2}{28} = \text{undefined}
\]

If \( x = 4 \),

\[
y = \frac{1}{4 - 5} + 5 - \frac{2}{4^2 + 3} = \frac{1}{-1} + 5 - \frac{2}{19} = \frac{1}{-1} + \frac{2}{19} = \text{undefined} - \frac{2}{19} = \text{undefined}
\]

Note: \( x^2 + 3 > 0 \)

Thus, if \( x = 4 \) or \( x = 5 \), \( y \) is undefined

If \( x \neq 4 \) and \( x \neq 5 \), then \( y \) is defined.

\[
\begin{align*}
y \text{ is defined} & \quad y \text{ is defined} & \quad y \text{ is defined} \\
4 & \quad 5
\end{align*}
\]

Domain of the function \( y = \frac{1}{x-5} + 5 - \frac{2}{x^2 + 3} \) is \( (-\infty, 4) \cup (4, 5) \cup (5, \infty) \).
Example 9:

Suppose \( y = \sqrt{x} \)

<table>
<thead>
<tr>
<th>x</th>
<th>y is undefined</th>
<th>y is defined</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-6</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-5</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-4</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-2</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>nonreal</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>y is defined</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>y is defined</td>
</tr>
<tr>
<td>2</td>
<td>1.414213562</td>
<td>y is defined</td>
</tr>
<tr>
<td>3</td>
<td>1.732050808</td>
<td>y is defined</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>y is defined</td>
</tr>
<tr>
<td>5</td>
<td>2.236067977</td>
<td>y is defined</td>
</tr>
</tbody>
</table>

If \( x < 0 \), then \( y \) is undefined.

If \( x \geq 0 \), then \( y \) is defined.

Domain of the function \( y = \sqrt{x} \) is \( [0, \infty) \).
Graph of $f(x) = \sqrt{x}$
Example 10:

Suppose \( y = \sqrt{x - 4} \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>nonreal</td>
</tr>
<tr>
<td>-1</td>
<td>nonreal</td>
</tr>
<tr>
<td>0</td>
<td>nonreal</td>
</tr>
<tr>
<td>1</td>
<td>nonreal</td>
</tr>
<tr>
<td>2</td>
<td>nonreal</td>
</tr>
<tr>
<td>3</td>
<td>nonreal</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>1.414213562</td>
</tr>
<tr>
<td>7</td>
<td>1.732050808</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>9</td>
<td>2.236067977</td>
</tr>
<tr>
<td>10</td>
<td>2.449489743</td>
</tr>
</tbody>
</table>

If \( x < 4 \), then \( y \) is undefined.

If \( x \geq 4 \), then \( y \) is defined.

Domain of the function \( y = \sqrt{x - 4} \) is \([4, \infty)\).

To find domain of square root functions, set the expression under the square root must be greater than or equal to zero.
Set $x - 4 \geq 0$
$x - 4 + 4 \geq 0 + 4$
$x \geq 4$
Example 11:

Find the domain of \( y = \frac{2}{\sqrt{x-4}} \).

In order for \( y \) to be defined or \( y \) to be a real number, the expression \( x - 4 > 0 \).

\[
\begin{align*}
x - 4 &> 0 \\
x - 4 + 4 &> 0 + 4 \\
x &> 4
\end{align*}
\]

Thus, \( y \) is defined if \( x > 4 \).

Domain of \( y = \frac{2}{\sqrt{x-4}} \) is \((4, \infty)\)

Set \( x - 4 > 0 \).

\[
\begin{align*}
x - 4 + 4 &> 0 + 4 \\
x &> 4
\end{align*}
\]
Graph of $f(x) = \frac{2}{\sqrt{x-4}}$
Example 12:

Find the domain of \( y = \sqrt{4x - 15} \).

In order for \( y \) to be defined or \( y \) to be a real number, the expression \( 4x - 15 \geq 0 \).

Thus, \( 4x - 15 \geq 0 \)
\[
4x - 15 + 15 \geq 0 + 15
\]
\[
4x \geq 15
\]
\[
\frac{4x}{4} \geq \frac{15}{4}
\]
\[
x \geq \frac{15}{4}
\]

Thus, \( y \) is defined if \( x \geq \frac{15}{4} \).

\[
\text{Domain of } y = \sqrt{4x - 15} \text{ is } \left[ \frac{15}{4}, \infty \right).
\]
Example 13:

Find the domain of \( y = \sqrt{20 - 4x} \).

In order for \( y \) to be defined or \( y \) to be a real number, the expression \( 20 - 4x \geq 0 \).

\[
\begin{align*}
20 - 4x & \geq 0 \\
20 - 4x - 20 & \geq 0 - 20 \\
-4x & \geq -20 \\
\frac{-4x}{-4} & \geq \frac{-20}{-4} \\
x & \leq 5
\end{align*}
\]

Thus, \( y \) is defined if \( x \leq 5 \).

\[
\begin{array}{c|c}
\text{y is defined} & \text{y is undefined} \\
\hline
\end{array}
\]

\hspace{1cm} 5

Domain of \( y = \sqrt{20 - 4x} \) is \((-\infty, 5] \).
Example 14:

Find the domain of \( y = \frac{\sqrt{2 - 4x}}{x + 2} \).

In order for \( y \) to be defined or \( y \) to be a real number, the expression \( 2 - 4x \geq 0 \) and \( x \neq -2 \).

\[
\begin{align*}
2 - 4x &\geq 0 \\
2 - 4x - 2 &\geq 0 - 2 \\
-4x &\geq -2 \\
\frac{-4x}{-4} &\geq \frac{-2}{-4} \\
x &\leq 0.5
\end{align*}
\]

Thus, \( y \) is defined if \( x \leq 0.5 \) and \( x \neq -2 \).

Thus, \( y \) is defined if \( x \leq 0.5 \) and \( x \neq -2 \).

\[
\text{Domain of } y = \frac{\sqrt{2 - 4x}}{x + 2} \text{ if } (-\infty, -2) \cup (-2, 0.5].
\]
Graph of \( f(x) = \sqrt{2-4x^2}/(x+2) \)
Combinations of Functions

Example 15:

Let \( f(x) = x^2 + 4 \) and \( g(x) = x + 4 \). Find \( f(x) + g(x) \).

\[
f(x) + g(x) = (x^2 + 4) + (x + 4) = x^2 + x + 8
\]

Domain of \( f(x) + g(x) = x^2 + x + 8 \) is \((-\infty, \infty)\)
Example 16:

Let \( f(x) = x^2 + 4 \) and \( g(x) = x + 4 \). Find \( f(x) - g(x) \).

\[
f(x) - g(x) = (x^2 + 4) - (x + 4) = x^2 + 4 - x - 4 = x^2 - x
\]

Domain of \( f(x) - g(x) = x^2 - x \) is \((-\infty, \infty)\)
Example 17:

Let \( f(x) = x^2 + 4 \) and \( g(x) = x + 4 \). Find \( f(x) \cdot g(x) \).

\[
f(x) \cdot g(x) = (x^2 + 4)(x + 4) = x^3 + 4x^2 + 4x + 16
\]

Domain of \( f(x) \cdot g(x) = x^3 + 4x^2 + 4x + 16 \) is \((-\infty, \infty)\)
Example 18:

Let \( f(x) = x^2 + 4 \) and \( g(x) = x + 4 \). Find \( \frac{f(x)}{g(x)} \) and \( \left( \frac{f}{g} \right)(3) \).

\[
\frac{f(x)}{g(x)} = \frac{x^2 + 4}{x + 4}
\]

Note that if \( x = -4 \), then \( \frac{f(x)}{g(x)} = \frac{x^2 + 4}{x + 4} = \frac{(-4)^2 + 4}{(-4) + 4} = \frac{-12}{0} = \text{undefined.} \)

Domain of \( \frac{f(x)}{g(x)} = \frac{x^2 + 4}{x + 4} \) is \((-\infty, -4) \cup (-4, \infty)\)

\[
\left( \frac{f}{g} \right)(3) = \frac{(3)^2 + 4}{(3) + 4} = \frac{13}{7}
\]
Example 19:

Let \( f(x) = \sqrt{x - 2} \) and \( g(x) = x + 1 \). Find \( \frac{f(x)}{g(x)} \).

\[
\frac{f(x)}{g(x)} = \frac{\sqrt{x - 2}}{x + 1}
\]

Note: \( \frac{f(x)}{g(x)} = \frac{\sqrt{x - 2}}{x + 1} \) is defined if \( x - 2 \geq 0 \) and \( x \neq -1 \).

\[
x - 2 \geq 0
\]
\[
x - 2 + 2 \geq 0 + 2
\]
\[
x \geq 2
\]

Domain of \( \frac{f(x)}{g(x)} = \frac{\sqrt{x - 2}}{x + 1} \) is \([2, \infty)\).
Example 20:

Let \( f(x) = \sqrt{x-2} \) and \( g(x) = \sqrt{x+1} \). Find \( f(x) + g(x) \).

\[
f(x) + g(x) = \sqrt{x-2} + \sqrt{x+1}
\]

Note: \( f(x) + g(x) = \sqrt{x-2} + \sqrt{x+1} \) is defined if \( x - 2 \geq 0 \) and \( x + 1 \geq 0 \).

Note: \( x - 2 \geq 0 \)

\[
x - 2 + 2 \geq 0 + 2 \\
x \geq 2
\]

Note: \( x + 1 \geq 0 \)

\[
x + 1 - 1 \geq 0 - 1 \\
x \geq -1
\]

Note: \( x \geq -1 \) and \( x \geq 2 \)
Domain of $f(x) + g(x) = \sqrt{x - 2} + \sqrt{x + 1}$ is $[2, \infty)$. 
Example 21:

Let \( f(x) = 3 + \frac{3}{x-1} \) and \( g(x) = \sqrt{x-6} \). Find \( g(x) - f(x) \).

\[
g(x) - f(x) = \sqrt{x-6} - \left(3 + \frac{3}{x-1}\right) = \sqrt{x-6} - 3 - \frac{3}{x-1}
\]

Note: \( g(x) - f(x) = \sqrt{x-6} - 3 - \frac{3}{x-1} \) is defined if \( x-6 \geq 0 \) and \( x \neq 1 \).

Note: \( x-6 \geq 0 \)
\[
x-6 + 6 \geq 0 + 6
\]
\[
x \geq 6
\]

\[
\text{y is undefined}
\]
\[
\text{y is defined}
\]

\[
\text{Domain of } g(x) - f(x) = \sqrt{x-6} - 3 - \frac{3}{x-1} \text{ is } [6, \infty)
\]
Graph of $f(x) = \sqrt{x-6} - 3 - 3(x-1)$
Composition of Functions

Example 22:

Let \( f(x) = x^2 + 4 \) and \( g(x) = x + 1 \)

Find \( (f \circ g)(x) \) and \( (f \circ g)(-2) \).

\[
(f \circ g)(x) = f(g(x)) \\
(f \circ g)(x) = f(x + 1) \\
(f \circ g)(x) = (x + 1)^2 + 4 \\
(f \circ g)(x) = x^2 + x + x + 1 + 4 \\
(f \circ g)(x) = x^2 + 2x + 5
\]

\[
(f \circ g)(-2) = (-2)^2 + 2(-2) + 5 = 4 - 4 + 5 = 5
\]

Domain of \( (f \circ g)(x) = x^2 + 2x + 5 \) is \((-\infty, \infty)\)
Graph of $f(x) = x^2 + 2x + 5$
Example 23:

Let \( f(x) = x^2 + 4 \) and \( g(x) = \sqrt{x+1} \)

Find \((g \circ f)(x)\) and \((g \circ f)(3)\).

\[
(g \circ f)(x) = g(f(x)) \\
(g \circ f)(x) = g(x^2 + 4) \\
(g \circ f)(x) = \sqrt{x^2 + 4} + 1 \\
(g \circ f)(x) = \sqrt{x^2 + 5}
\]

\[
(g \circ f)(3) = \sqrt{(3)^2 + 5} = \sqrt{14}
\]

\( (g \circ f)(x) = \sqrt{x^2 + 5} \) is defined if \( x^2 + 5 \geq 0 \).

Since \( x^2 + 5 \) is always greater than 0, domain of \((g \circ f)(x) = \sqrt{x^2 + 5} \) is \((-\infty, \infty)\)
Example 24:

Let \( f(x) = \frac{x}{x-2} \) and \( g(x) = \frac{3}{x} \)

Find \( (g \circ f)(x) \) and \( (g \circ f)(3) \).

\[
(g \circ f)(x) = g(f(x))
\]

\[
(g \circ f)(x) = g\left(\frac{x}{x-2}\right)
\]

\[
(g \circ f)(x) = \frac{3}{x} \quad x \neq 2
\]

\[
(g \circ f)(x) = \frac{3}{x} \quad x \neq 2
\]

\[
(g \circ f)(x) = \frac{3}{x} = \frac{3}{1 \cdot x} = \frac{3x - 6}{x} \quad x \neq 0
\]

\[
(g \circ f)(x) = \frac{3x - 6}{x} \quad \text{is defined if } x \neq 0 \text{ and } x \neq 2.
\]

\[
\text{Domain of } (g \circ f)(x) = \frac{3x - 6}{x} \text{ is } (-\infty, 0) \cup (0, 2) \cup (2, \infty)
\]
Graph of $f(x) = 3(x(x-2))$
Find Function Values Using Graphs

Use graphs to find the following:

a) \( f(2) + g(0) \)
b) \( f(2) - g(0) \)
c) \( f(2) \cdot g(0) \)
d) \( \frac{f(2)}{g(2)} \)
e) \( (f \circ g)(-2) \)
f) \( (f \circ g)(0) \)

a) \( f(2) = 1 \) Using point (2, 1) on orange graph

\( g(0) = 6 \) Using point (0, 6) on green graph

Thus, \( f(2) + g(0) = 1 + 6 = 7 \)
b) \( f(2) = 1 \)  Using point (2, 1) on orange graph
\( g(0) = 6 \)  Using point (0, 6) on green graph

Thus, \( f(2) - g(0) = 1 - 6 = -5 \)

c) \( f(2) = 1 \)  Using point (2, 1) on orange graph
\( g(0) = 6 \)  Using point (0, 6) on green graph

Thus, \( f(2) \cdot g(0) = (1)(6) = 6 \)

d) \( f(2) = 1 \)  Using point (2, 1) on orange graph
\( g(0) = 6 \)  Using point (0, 6) on green graph

Thus, \( f(2) / g(0) = 1/6 \)

e) \( (f \circ g)(-2) = f(g(-2)) \)

Note: \( g(-2) = 0 \)  Using point (-2, 0) on green graph

Hence, \( (f \circ g)(-2) = f(g(-2)) \)
\( (f \circ g)(-2) = f(0) \)

Note: \( f(0) = 2 \)  Using point (0, 2) on orange graph

Thus, \( (f \circ g)(-2) = f(0) = 2 \)
f) \((g \circ f)(0) = g(f(0))\)

Note: \(f(0) = 2\) Using point \((0, 2)\) on orange graph

Hence, \((g \circ f)(0) = g(f(0))\)
\[(g \circ f)(-2) = g(2)\]

Note: \(g(2) = 10\) Using point \((2, 10)\) on green graph

Thus, \((f \circ g)(-2) = f(0) = 2\)