Solving Linear Inequalities

Set Notation:

$(2, 10) = \text{all real numbers between 2 and 10, not including 2 or 10}$
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\((2, 10]\) = all real numbers between 2 and 10, including 10 but not 2
Example 1:

Find all values that would satisfy the following inequality.

\[ x \leq 7 \]

The solution set is a set of real numbers that would satisfy this inequality.

A solution of this inequality is any real number less than or equal to 7.

Hence, the solution set is set of real numbers less than or equal to 7.

Solution set is \((-\infty, 7]\).

Solution set is \(\{x \mid x \leq 7\}\).
Example 2:

Find all values that would satisfy the following inequality.

\[ x > -17 \]

The solution set is a set of real numbers that would satisfy this inequality.

A solution of this inequality is any real number greater than -17.

Hence, the solution set is set of real numbers greater than -17. Solution set is \((-17, \infty)\).

Solution set is \(\{x \mid x > -17\}\).
Example 3:

Find all values that would satisfy the following inequality.

\[ x + 5 \leq 17 \]

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

\[
\begin{align*}
x + 5 & \leq 17 \\
x + 5 - 5 & \leq 17 - 5 \\
x & \leq 12
\end{align*}
\]

Hence, set of all real numbers that satisfy the inequality \( x + 5 \leq 17 \) includes all real numbers less than or equal to 12.

Solution set is \((-\infty, 12]\).
Solution set is \( \{x \mid x \leq 12\} \).
Example 4:

Find all values that would satisfy the following inequality.

\[ 2x + 5 \leq 16 \]

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

\[
\begin{align*}
2x + 5 & \leq 16 \\
2x + 5 - 5 & \leq 16 - 5 \\
2x & \leq 11 \\
\frac{2x}{2} & \leq \frac{11}{2} \\
x & \leq \frac{11}{2}
\end{align*}
\]

Hence, set of all real numbers that satisfy the inequality \( 2x + 5 \leq 16 \) includes all real numbers less than or equal to \( \frac{11}{2} \).

Solution set is \( (-\infty, \frac{11}{2}] \Leftrightarrow \left\{ x \mid x \leq \frac{11}{2} \right\} \).
Example 5:

Find all values that would satisfy the following inequality.
\[ 2x + 5 \leq 8x + 16 \]

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

\[ 2x + 5 \leq 8x + 16 \]
\[ 2x + 5 - 5 \leq 8x + 16 - 5 \]
\[ 2x \leq 8x + 11 \]
\[ 2x - 8x \leq 8x - 8x + 11 \]
\[ -6x \leq 11 \]
\[ -6 \leq -6 \]
\[ x \geq -\frac{11}{6} \]  
Reverse inequality sign when dividing negative number.

Hence, set of all real numbers that satisfy the inequality \( 2x + 5 \leq 8x + 16 \) includes all real numbers greater than or equal to \( -\frac{11}{6} \).

Solution set is \( \left( -\frac{11}{6}, \infty \right) \)

Solution set is \( \left\{ x \mid x \geq -\frac{11}{6} \right\} \).
Example 6:

Find all values that would satisfy the following inequality.

\[
\frac{2x}{3} + 5 \leq \frac{8x}{5} + \frac{11}{4}
\]

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

\[
\frac{2x}{3} + 5 \leq \frac{8x}{5} + \frac{11}{4}
\]

\[
\frac{2x}{3}(60) + 5(60) \leq \frac{8x}{5}(60) + \frac{11}{4}(60)
\]

\[40x + 300 \leq 96x + 165\]

\[40x + 300 - 300 \leq 96x + 165 - 300\]

\[40x \leq 96x - 135\]

\[40x - 96x \leq 96x - 96x - 135\]

\[-56x \leq -135\]

\[-\frac{56x}{-56} \leq \frac{-135}{-56}\]

\[x \geq \frac{135}{56}\]

Reverse inequality sign when dividing negative number.

Hence, set of all real numbers that satisfy the inequality \(\frac{2x}{3} + 5 \leq \frac{8x}{5} + \frac{11}{4}\)

includes all real numbers greater than or equal to \(\frac{135}{56}\).
Solution set is \( \left( \frac{135}{56}, \infty \right) \).

Solution set is \( \left\{ x \mid x \geq \frac{135}{56} \right\} \).

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Example 7:

Find all values that would satisfy the following inequality.

\[
\frac{x - 4}{3} + 5 \leq \frac{x + 3}{5} + \frac{3}{10}
\]

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

\[
\begin{align*}
\frac{x - 4}{3} + 5 & \leq \frac{x + 3}{5} + \frac{3}{10} \\
\frac{(x - 4)}{3} + 5 & \leq \frac{(x + 3)}{5} + \frac{3}{10} \\
(30)\frac{(x - 4)}{3} + 5(30) & \leq (30)\frac{(x + 3)}{5} + \frac{3}{10}(30) \\
10(x - 4) + 150 & \leq 6(x + 3) + 9 \\
10x - 40 + 150 & \leq 6x + 18 + 9 \\
10x + 110 & \leq 6x + 27 \\
10x + 110 - 110 & \leq 6x + 27 - 110 \\
10x & \leq 6x - 83 \\
10x - 6x & \leq 6x - 6x - 83 \\
4x & \leq -83 \\
\frac{4x}{4} & \leq \frac{-83}{4} \\
x & \leq \frac{-83}{4}
\end{align*}
\]
Hence, set of all real numbers that satisfy the inequality \( \frac{x-4}{3} + 5 \leq \frac{x+3}{5} + \frac{3}{10} \)

includes all real numbers less than or equal to \( \frac{-83}{4} \).

Solution set is \( (-\infty, \frac{-83}{4}] \).

Solution set is \( \{ x \mid x \leq \frac{-83}{4} \} \).
Example 8:

Find all values that would satisfy the following inequality.

$$3(x - 4) - (x - 10) > -5(x + 2)$$

To find the set of real numbers that would satisfy this inequality, we would proceed as we do when solving equations.

$$3(x - 4) - (x - 10) > -5(x + 2)$$
$$3(x - 4) - 1(x - 10) > -5(x + 2)$$
$$3x - 12 - 1x + 10 > -5x - 10$$
$$2x - 2 > -5x - 10$$
$$2x - 2 + 2 > -5x - 10 + 2$$
$$2x > -5x - 8$$
$$2x + 5x > -5x + 5x - 8$$
$$7x > -8$$
$$\frac{7x}{7} > \frac{-8}{7}$$
$$x > \frac{-8}{7}$$

Hence, set of all real numbers that satisfy the inequality $3(x - 4) - (x - 10) > -5(x + 2)$ includes all real numbers greater than $\frac{-8}{7}$. 
Solution set is \(\left(-\frac{8}{7}, \infty\right)\).

Solution set is \(\left\{x \mid x > -\frac{8}{7}\right\}\).